K will precede

THEOREMATUM IN LIBRIS ARCHIMEDIS DE SPHAERA & CYLINDRO Declaratio.

Authore
GUILELMO OUGHTREDO
ANGLO.

OXONIÆ,
Excudebat LEON. LICHFIELD, Veneunt apud THO. ROBINSON. *Anno Dom.* 1652.

Rerum quarundam denotationes.

R radius, eſt ſemidiameter circuli, ſive uno conſtet nomine AO, vel Eω, vel IU : ſive duobus ut AO+Eω, vel Eω+IU : ut in ſchemate 1.

$\delta . \pi :: $ ſemidiameter. ſemiperipheria.

$\frac{\pi}{\delta}$R, eſt ſemiperipheria circuli cujus Radius eſt R.

$\frac{\pi}{\delta}$:AO+Eω: eſt ſemiperipheria circuli cujus Radius eſt AO+Eω.

$\frac{\pi}{\delta}$Rq, eſt area circuli.

$\frac{\pi}{3\delta}$Rq $\times$ altitud: vel $\frac{\pi}{\delta}$Rq in $\frac{1}{3}$ Altitud, eſt Conus; ſcil: $\frac{1}{3}$ Cylindri.

$\frown$ ſignificat ſuperficiem curvam.

Coni & Cylindri, qui in æqualibus ſunt baſibus, ſunt ut altitudines. 14 e 12.

Æqualium Conorum & Cylindrorum baſes & altitudines reciprocantur. 15 e 12.

Aſſumo, Figuram regularem infinitorum laterum, cui nec major inſcribi, nec minor circumſcribi poterit; ſi plana ſit, eſſe circulum; ſin ſolida, eſſe ſphæram.

Theorematum

Theorematum in Libris ARCHIMEDIS de Sphæra & Cylindro Declaratio.

DUODECIM primas propositiones, quia demonstrationibus negativis, quas ego ut parùm scientificas, quantum possum, evito, inque ipsarum loco affirmativas substituo, inserviunt, missas faciam.

I. In Cylindro recto, Si 2R, M, Latus; hoc est, $2AO, M, KA, \div\div$: Dico $\frac{\pi}{\delta}$ $Mq = \frown$ Cylindri.

Nam $\frac{\pi}{\delta}$ $Mq = \frac{\pi}{\delta}$ $2AO \times KA$. 13 l 1.

(Ad septem theoremata sequentia pertinet schema I.)

II. In Cono æquicruro KON, si $KO, M, AO \div\div$: Dico $\frac{\pi}{\delta}$ $Mq = \frown$ Coni. Nam $\frac{\pi}{\delta}$ $Mq = \frac{\pi}{\delta}$ AO in KO. 14 l 1.

 III In

III. In Cono æquicruro KON, Dico esse semid: basis. Latus :: Basis. $\frown$ Coni. Nam AO. KO :: $\frac{\pi}{\delta}$ AOq. $\frac{\pi}{\delta}$AO in KO. 15 l 1.

IV. In Cono æquicruro KON, Si AO+Eω, M, Oω=KO−Kω ∺: Dico $\frown$ frusti O$\omega\nu$N$=(\frac{\pi}{\delta}$Mq$)\frac{\pi}{\delta}$: AO+Eω: ×Oω. Nam per 2, $\frown$ O$\omega\nu$N$=\frac{\pi}{\delta}$: AO×KO: mi $\frac{\pi}{\delta}$: Eω×Kω: $=\frac{\pi}{\delta}$: AO+Eω : in: KO−Kω. Est enim AO+Eω in KO−Kω=AO × KO −Eω×Kω pl Eω × KO −AO×KO, quæ se invicem tollunt: Quia AO.Eω:: KO. Kω.

V. In Cono æquicruro KON, Si KO, M, AO ∺; & AP perpendicularis lateri KO: Dico $(\frac{\pi}{3\delta}$ Mq$)$ $\frac{\pi}{3\delta}$ AO×KO in AP$=\frac{\pi}{3\delta}$ AOq in KA=KON. Nam KA. AP :: KO. AO :: AO × KO. AOq. Ergo. 17 l 1.

VI. In Cono æquicruro KON, Si Kω. M. Eω ∺; & AP perpend: lateri KO: Dico $(\frac{\pi}{3\delta}$ Mq$)$ $\frac{\pi}{3\delta}$ Eω ×Kω in AP$=\frac{\pi}{3\delta}$-Eωq×KA, scil: rhombo KωAν. Nam KA. AP :: Kω. Eω :: Eω × Kω. Eωq. Ergo. 18 l 1.

VII. In Cono æquicruro KON, Dico frustum

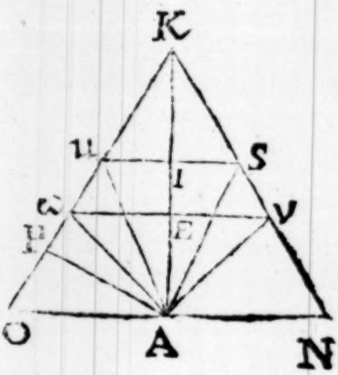
Conicè

Conicè excavatum $O\omega A\nu N$, æquari Cono cujus basis est æqualis $\bigcirc$ frusti $O\omega\nu N$, & altitudo AP: hoc est, con: KON — rhomb: $K\omega A\nu = \frac{\pi}{\delta}: AO+E\omega: \times O\omega$ in $\frac{1}{3}AP$. Nam

$$\left.\begin{array}{l} \frac{\pi}{\delta}\, AO \times KO \text{ in } \tfrac{1}{3}AP = KON, \text{per } 5 \\ \text{Et } \frac{\pi}{\delta}\, E\omega \times K\omega \text{ in } \tfrac{1}{3}AP = \text{rhomb: } K\omega A\nu, \text{ per } 6 \end{array}\right\} \text{ho-}$$

rum differentia est $\frac{\pi}{\delta}\, AO \times KO - \frac{\pi}{\delta}\, E\omega \times K\omega$, per 4, $= \frac{\pi}{\delta}: AO+E\omega: \times O\omega = \bigcirc\, O\omega A\nu$; ductis omnibus in $\frac{1}{3}AP$. Ergo, &c.

VIII. In Cono æquicruro KON, Dico rhombum conicè excavatum $\omega UAS\nu$, æquari Cono cujus basis est æqualis $\bigcirc$ frusti $\omega US\nu$, & altitudo AP: hoc est, rhomb $K\omega A\nu$-rhomb $KUAS = \frac{\pi}{\delta}: E\omega + IU: \times \omega U$ in $\frac{1}{3}AP$. Nam



$$\left.\begin{array}{l} \text{per } 6,\ \frac{\pi}{\delta}\, E\omega \times K\omega \text{ in } \tfrac{1}{3}AP = \text{rhomb } K\omega A\nu \\ \text{Et } \frac{\pi}{\delta}\, IU \times KU \text{ in } \tfrac{1}{3}AP = \text{rhomb } KUAS \end{array}\right\} \text{horum}$$

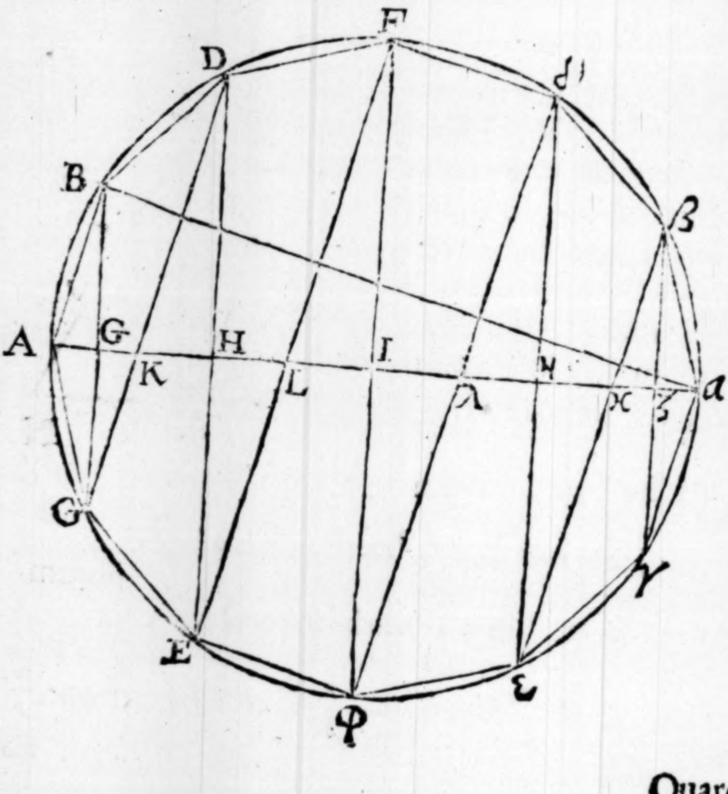
differentia est $\frac{\pi}{\delta}\, E\omega \times K\omega - \frac{\pi}{\delta}\, IU \times KU$, per 4, $= \frac{\pi}{\delta}:$

$E\omega + IU : \times\omega U = \bigcirc\,\omega US\nu$; ductis omnibus in $\frac{1}{2}$ AP. Ergo, &c.

IX. Si figura plana polygona laterum æqualium & numero parium. $ABDF\delta\beta\alpha\gamma\varepsilon\varphi EC$, inſcribatur circulo, junganturque anguli rectis lineis parallelis: Dico $AB.\ B\alpha :: A\alpha.\ BC + DE + F\varphi + \delta\varepsilon + \beta\gamma$; hoc eſt, $2BC + 2DE + F\varphi$. Nam $AB.\ B\alpha :: \frac{1}{2}AK.\ \frac{1}{2}BC :: \frac{1}{2}KL.\ \frac{1}{2}DE :: \frac{1}{2}L\lambda.\ \frac{1}{2}F\varphi :: \frac{1}{2}\lambda\varkappa.\ \frac{1}{2}\delta\varepsilon :: \frac{1}{2}\varkappa\alpha.\ \frac{1}{2}\beta\gamma$.

Quare $A\alpha \times B\alpha = AB$ in $2BC + 2DE + L\varphi$. 21 l 1.

Et in ſegmento $A\delta\varepsilon$, erit $AB.\ B\alpha :: A\eta.\ BC + DE + F\varphi + \frac{1}{2}\delta\varepsilon$.



Quare

Quare $A\alpha \times B\alpha = AB$ in $BC + DE + F\varphi + \frac{1}{2}\delta\varepsilon$.

X. Si circulo, vel circuli ſegmento alicui figura ejuſmodi plana polygona laterum æqualium & parium, tum inſcribatur, tum circumſcribatur; & diametro $A\alpha$ quieſcente, circulus circumvolvatur; deſcribetur figura ſolida conſtans ſuperficiebus quibuſdam Conicis: Et paralleli BC, DE, $F\varphi$, $\delta\varepsilon$, $\beta\gamma$, deſcribent totidem circulos parallelos. Atque in his, quæ circumſcripta eſt, ſive continens, major ſemper eſt circulo incluſo: & quæ inſcripta eſt, minor ſemper erit circulo ambiente. Et ſuperficies figuræ circumſcriptæ, ad ſuperficiem figuræ inſcriptæ ſimilis, eſt in ratione laterum duplicata: At figura ipſa ſolida circumſcripta, ad ſolidam ſimilem inſcriptam, in ratione triplicata. 22. 27. 30. 34. 37 l 1.

XI. Si diameter circuli includentis ejuſmodi figuram ſolidam, ſit $A\alpha$: fiatque $A\alpha$, M, $B\alpha$ ∺: vel, quod idem eſt, per 9, $2BC+2DE+F\varphi$, M, AB ∺: Dico $\frac{\pi}{\delta}\,Mq$ = ſuperficiei figuræ. Nam per 2, $\frac{\pi}{\delta}\,BC \times AB$

$= 2$ ◠ coni ABC: & per 4, $\frac{\pi}{\delta}\,BC+DE$ in $AB = 2$ ◠

fruſti $BCED$: & $\frac{\pi}{\delta}\,DE+F\varphi$ in $AB = 2$ ◠ fruſti $DE\varphi F$.

Ergo $\frac{\pi}{\delta}\,2BC+2DE+F\varphi$ in $AB =$ ◠ figuræ totius,

nempe $\frac{\pi}{\delta}\,Mq$. 23. 28 l 1.

XII. In Schem: 3. Figuræ ejuſmodi ſolidæ, ſi

ſphæræ

ſphæræ inſcribatur, ſuperficies $\frac{\pi}{\delta}Mq$ minor eſt circulo habente axem ſphæræ continentis $A\alpha$ pro diametro. Nam $M \sqsubset A\alpha$.

Sin circumſcribatur, ſuperficies $\frac{\pi}{\delta}$ Mq major eſt circulo habente axem ſphæræ contentæ $2IP = B\alpha$ pro diametro. Nam $A\alpha$, M, 2IP $\div\!:$ Quare M cadet inter A & Q. 24, 29 l 1.

XIII. Quidni igitur ſphæræ ſuperficies æquetur quatuor maximis circulis; nempe $\frac{\pi}{\delta}$ Diam:q? 31 l 1.

XIV. Figura ejuſmodi ſolida æqualis eſt Cono, cujus Baſis eſt circulus æqualis ſuperficiei figuræ; & Altitudo IP perpend: è centro ſphæræ in latus figuræ: hoc eſt, per 11, $\frac{\pi}{3\delta}$ Mq in (IP) $\frac{1}{2}B\alpha$ = figuræ toti ſolidæ. Nam per 6, Rhomb: $BACI = \frac{1}{3} \frown BAC$ in IP. Et per 8, Excavatum $DBICE = \frac{1}{3} \frown DBCE$ in IP. Et per 7, Excavatum $FDIE\varphi = \frac{1}{3} \frown FDE\varphi$ in IP. Et ſimiliter pro altero hæmiſphærio. Quare $\frac{1}{3} \frown BAC + \frac{1}{3} \frown DBCE + \frac{1}{3} \frown FDE\varphi$ in $B\alpha$ (2IP) = toti figuræ ſolidæ; nempe $\frac{\pi}{3\delta}$ Mq: vel $\frac{\pi}{3\delta}$ $A\alpha \times B\alpha$ in $\frac{1}{2}$BA(IP). 25, 29 l 1.

XV. Figura ejuſmodi, ſi ſphæræ inſcribatur, minor eſt quatuor Conis habentibus baſem æqualem circulo ſphæræ maximo: hoc eſt Cono habenti baſem æqualem ſuperficiei ſphæræ; altitudinem verò æqualem ſemiaxi. Sin circumſcribatur, iiſdem major eſt.

Nam

Nam per 12, superficies figuræ inscriptæ, superficie sphæræ minor est: circumscriptæ autem, major. 26. 29 l 1.

XVI. Quidni igitur ipsa sphæra æqualis sit quatuor Conis habentibus basem æqualem circulo sphæræ maximo; hoc est Cono habenti basem æqualem superficiei sphæræ; Altitudinem verò æqualem semiaxi? 32 l 1.

Consect. $\frac{2}{3}$ Cylind: $=$ Sphæræ $=$ 2 Conis. Nam $\frac{\pi}{3\delta} Rq \times 4R =$ Sphæræ. Et $\frac{\pi}{\delta} Rq \times 2R =$ Cylindro. $\frac{\pi}{3\delta} Rq \times 2R =$ Cono.

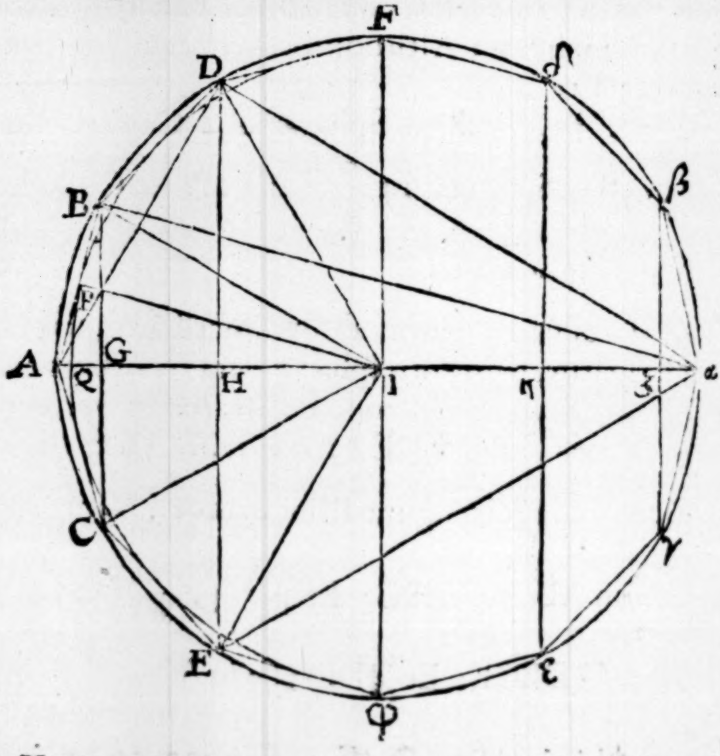
XVII. Si figura ejusmodi sive inscribatur, sive circumscribatur, segmento sphæræ, puta $A\delta\varepsilon$, cujus basis sit $\delta\varepsilon$; altitudo $A\eta$; fiatque $B\alpha$, M, $A\eta$ ∺: vel, quod idem est, per 9, $BC + DE + F\varphi + \frac{1}{2}\delta\varepsilon$, M, AB ∺: Dico $\frac{\pi}{\delta} Mq =$ superficiei figuræ illius mancæ. Nam per 2, $\frac{\pi}{\delta} \frac{1}{2}BC$ in $AB =$ ◠ ABC. Et per 4, $\frac{\pi}{\delta} \frac{1}{2}BC + \frac{1}{2}DE$ in $AB =$ ◠ $DBCE$: Et $\frac{\pi}{\delta}\frac{1}{2}DE + \frac{1}{2}F\varphi$ in $AB =$ ◠ $FDE\varphi$. Et $\frac{\pi}{\delta}\frac{1}{2}F\varphi + \frac{1}{2}\delta\varepsilon$ in $AB =$ ◠ $\delta F\varphi\varepsilon$. Ergo 33. 37 l 1.

XVIII. Figuræ ejusmodi mancæ, si segmento sphæræ, puta $A\delta\varepsilon$, inscribatur, superficies $\frac{\pi}{\delta} Mq \sqsubset \frac{\pi}{\delta} A\delta q$. Nam $A\delta q = A\alpha \times A\eta \sqsubset B\alpha \times A\eta$.

Sin

Sin circumſcribatur, ſuperficies $\frac{\pi}{\delta}$ Mq ⊏ $\frac{\pi}{\delta}$ Aδq.

Nam Bα=2IQ eſt diameter ſphæræ interioris ſive contentæ. Eſtque Aη ⊏ Qη. Quare $\frac{1}{2}$ M protenditur ultra IQ diametrum ſphæræ contentæ. 35.38.41 l 1.



XIX. Quidni igitur ipſa ſuperficies ſegmenti ſphæræ æqualis ſit circulo, cujus ſemidiameter eſt recta ducta a vertice ſegmenti in finem baſis? 40 l 1.

XX. Figura ejuſmodi manca, ſive inſcribatur ſegmento ſphæræ, puta DAE minori ſemicirculo, vel DαE majori, æqualis eſt Cono habenti baſem æqualem ſuperficiei illius figuræ mancæ; altitudinem

verò

verò æqualem perpend: IP è centro in latus figuræ, adjecto vel ablato Cono DIE medio; hoc eſt, toti ſolido DBACEI, vel reliquo DF$\delta\beta\alpha\gamma\varepsilon\varphi$EI. Nam in ſolido minore DBA=CEI, per 6, Rhomb: BACI $= \frac{1}{3}$ ∩ BAC in IP. Et per 8, $\frac{1}{3}$ ∩ DBCE in IP = excavato DBICE. Ergo. Et ſimiliter de majore DF$\delta\beta\alpha\gamma\varepsilon\varphi$EI. 36. 39 l 1.

Conſect. Quare & per 18, Si figura ejuſmodi ſegmento ſphæræ, puta DAE, vel DαE, inſcribatur, totum ſolidum minus eſt Cono, cujus baſis eſt circulus a ſemidiametro AD, vel αD; & altitudo ſemiaxis IA.

XXI. Quidni igitur Sector ſphæræ DBACEI, æqualis ſit Cono $\frac{\varpi}{3\delta}$ ADq in IA; & reliquus Dα=EI, æqualis Cono $\frac{\varpi}{3\delta}$ αDq in IA? 42 l 1.

XXII. Si fiat Hα. Hα+Iα:: HA. HS: Dico $\frac{\varpi}{3\delta}$ DHq in HS = ſegm: ſphæræ DAE. Nam Hα. HA :: Hα + Iα − Hα. HS − HA :: Iα. AS. Eſtque (Iα + AS) IS. IA :: (Hα + HA) Aα. Hα :: Aαq. αDq :: ADq. DHq. Quare $\frac{\varpi}{3\delta}$ DHq × IS = $\frac{\varpi}{3\delta}$ ADq × IA = ſegmento DAE + (Cono DIE) $\frac{\varpi}{3\delta}$ DHq×IH.

Ergo